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General models for optimum tilt angles of solar panels: Turkey case study

Kadir Bakirci *

Department of Mechanical Engineering, University of Atatürk, 25240 Erzurum, Turkey

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ABSTRACT

This paper deals with finding the optimum tilt angle of solar panels for solar energy applications. The optimization of tilt angles was performed using solar radiation data measured for eight big provinces in Turkey. The optimum angle for tilted surfaces varying from 0° to 90° in steps of 1° was calculated by searching for the values of which the daily total solar radiation was at a maximum for a specific period. It was found that the optimum tilt angle changed between 0° and 65° throughout the year in Turkey. It was seen that the optimum tilt angle reached a minimum of 0° in June and July and, the monthly average daily total radiation at this angle was generally at a maximum. In addition, the optimum tilt angle increased during the winter months and reached a maximum in December in all provinces. Likewise, general correlations were developed to estimate the optimum tilt angle of solar collectors used in Turkey and their accuracies were compared on the basis of statistical error tests of Mean Bias Error (MBE), Root Mean Square Error (RMSE), *t*-statistic (*t*-stat) and correlation coefficient (*r*).

Contents

1. Introduction

Turkey is an energy importing nation with more than half of our energy requirements met by imported fuels. Air pollution is becoming a significant environmental concern in the country. Achieving solutions to the environmental problems that we face today requires long-term potential actions for sustainable development. In this regard, renewable energy resources appear to be the one of the most efficient and effective solutions. For governments or societies to attain sustainable development, a great deal of effort must be devoted to utilizing sustainable energy resources in terms of renewable energies [1,2]. The studies on the new and renewable energy sources have gained speed and are encouraged due to the fact that energy resources used today run out rapidly and cause environmental pollution [3].

Estimates of the monthly average solar radiation incident on the surface of various orientations are required for solar energy design procedures. Monthly averages of the daily solar radiation incident upon a horizontal surface are available in the literature for many locations [4–9]. However, radiation data on tilted surfaces is not generally available. Tiris et al. [5] calculated the correlations of the monthly average daily global, diffuse and beam radiations with hours of bright sunshine in Gebze, Turkey. Bakirci [7] developed the correlations for estimation of daily global solar

* Fax: +90 442 2360957.
E-mail address: abakirci@atauni.edu.tr

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Nomenclature

H	Monthly average daily global radiation on horizontal surface (MJm^{-2}-day)
H_d	Monthly average daily diffuse radiation on horizontal surface (MJm^{-2}-day)
H_o	Monthly average daily extraterrestrial radiation (MJm^{-2}-day)
H_T	Monthly average daily global radiation on tilted surface (MJm^{-2}-day)
K_T	Clearness index
R_b	Ratio of average beam radiation on a tilted surface to that on horizontal surface
s	Tilt of surface from horizontal
S_{opt}	Optimum tilt angle (°)
δ	Solar declination (°)
φ	Latitude of the place (°)
ρ	Ground reflectance
ω_s	Hour angle (°)
ω_s'	Sunset hour angle (°)

radiation with hours of bright sunshine in Turkey. Zuhairy and Sayigh [8] carried out simulation and modeling of solar radiation in Saudi Arabia. Ulgen and Hepbasli [9] investigated the diffuse fraction of daily and monthly global radiation for Izmir, Turkey.

The amount of energy collected by a solar collector and thus converted to thermal energy is highly influenced by its orientation and tilt angle. Designers of solar systems that use flat-plate collectors need to know the amount of solar energy collected by a tilted surface. For such a system, the collector slope is one of many design considerations. Calculations of the solar energy absorbed by a tilted surface directly facing the equator with an azimuth are available in the literature. However, designers often need to know how much energy can be absorbed by the surfaces with other azimuths. It is generally known that in the northern hemisphere, the optimum collector orientation is south-facing, and the optimum tilt depends upon the latitude and the day of the year [10].

Data of the monthly global and diffuse solar radiation obtained from observations over many years are required for acceptable estimation of the optimal tilt angle of a solar collector. However, the monthly diffuse radiation is not always available in many locations in Turkey. Therefore, in this article an empirical correlation suggested by Page [11] was used for the calculation of monthly diffuse radiation in the provinces considered and a simple mathematical procedure was also applied to estimate the optimal tilt angle of a solar collector.

The main objective of this study is to determine the optimal tilt angle of solar panels used in Turkey for maximizing their energy collection and to obtain general correlations for estimating the optimum slopes that are precise and convenient for use.

2. Background and previous works

An important parameter in the optimum utilization of solar panels is their tilt angle with the horizontal. This changes the amount of solar radiation that reaches the solar panels. The optimal tilt angle of a solar collector is related to the local climatic condition, the geographic latitude and the period of its use. Therefore, different locations will have different optimal tilt angles for solar collectors.

A number of studies on the optimal tilt angle of solar panels around the world have been presented [12–36]. However, studies carried out in Turkey are few in number and, the general correlations giving optimum slopes for Turkey are not obtained. Iqbal [12] investigated the optimum collector slope for residential heating in adverse climates. Soulayman [15] carried out a study on the optimum tilt of solar absorber plates. Hussein et al. [21] developed a theoretical analysis of the instantaneous, daily, and yearly enhancement in solar energy collection of a tilted flat-plate solar collector augmented by a plane reflector. Bari [22] described a method to determine the optimum slope angle and orientation of solar collectors for different periods of operation at any position in the Malaysian territory. The method utilizes both the direct and diffuse components of solar radiation. Tang and Wu [25] carried out a reasonable estimation of the optimal tilt angle of a fixed collector for maximizing its energy collection based on the monthly global and diffuse radiation on a horizontal surface. Gunerhan and Hepbasli [29] calculated the optimum tilt angles by searching for the values for which the total radiation on the collector surface is at a maximum for a particular day or a specific period.

Mehleri et al. [31] carried out a study on the determination of the optimum tilt angle and orientation for solar photovoltaic arrays in order to maximize the incident of solar irradiance exposed on the array, for a specific period of time. Moghadam et al. [32] performed optimization of solar flat collector inclination. Monthly, seasonal, semi-annual and annual optimum tilt angles were determined. Ghosh et al. [33] determined the seasonal optimum tilt angles, solar radiations on variously oriented, single and double axis tracking surfaces at Dhaka. Three mathematical models for the point source with parameters optimized for a variety of climatic conditions were employed to determine hourly and seasonal optimum tilt angles. Maatallah et al. [34] presented an overview on research works on solar radiation basics and photovoltaic generation. The effects of azimuth and tilt angles on the output power of a photovoltaic module were investigated. Kaldellis and Zafirakis [35] carried out an experimental study in the area of Athens in order to evaluate the performance of different PV panel tilt angles during the summer period. The angle of 15° (±2.5°) was designated as optimum for almost the entire summer period. Benghanem [36] performed a study on the optimum slope and orientation of a surface receiving a maximum solar radiation. The annual optimum tilt angle was found to be approximately equal to the latitude of the location. Siraki and Pillay [37] proposed a simple method on a modified sky model to calculate the optimum angle of installation for urban applications. It was expressed that the results demonstrated the dependency of the optimum angle of installation on the latitude, weather conditions and surroundings.

Lave and Kleissl [38] calculated the optimum tilt and azimuth angles of solar panels for a grid of 0.1° by 0.1° National Solar Radiation Database cells covering the continental United States. The yearly global irradiation incident on a panel at this optimum orientation was compared to the solar radiation received by a flat horizontal panel and a 2-axis tracking panel. Lubitz [39] investigated the effect of manual tilt adjustments on incident irradiance on fixed and tracking solar panels. The optimum tilt angle for an azimuth tracking panel was found to be on average 19° closer to the vertical than the optimum tilt angle for a fixed, south-facing panel at the same site. Chau [40] performed a study on optimum tilt angles for solar collectors in clear sky conditions. The optimum tilt angles for collectors at various latitudes were determined for all 12 months of the year. It was expressed that these results were also applicable to collectors with a cylindrically curved transparent cover. Moon et al. [41] determined the optimum tilt angles for solar collectors in 171 locations in North America. The equations that predicted the optimum collector tilt angle were derived by using regression analysis based on the results obtained for the 171 locations for several different periods of the year.

3. Estimation of solar radiation on an inclined surface

The monthly average values of solar radiation incident on surfaces of various orientations are required for solar energy applications. The monthly averages of the daily solar radiation incident upon a horizontal surface are available for many locations. However, radiation data on tilted surfaces are generally not available. A simple method to estimate the average daily radiation for each calendar month on surfaces facing directly towards the equator has been developed by Liu and Jordan [42].

The monthly average daily radiation on a horizontal surface (H), for each calendar month can be expressed by defining K_T, the fraction of the mean daily extraterrestrial radiation (H_o)

$$K_T = H/H_o \tag{1}$$

$$H_o = \frac{24}{\pi} I_{sc}\left(1+0.033\cos\frac{360n}{365}\right)\left(\cos\varphi\cos\delta\sin\omega_s + \frac{\pi\omega_s}{180}\sin\varphi\sin\delta\right) \tag{2}$$

where I_{sc} is the solar constant ($=1353$ W/m^2), n is the day of the year given for each month, φ is the latitude, δ is the solar declination and ω_s is the hour angle. Values of δ and ω_s are given by [43,44].

$$\delta = 23.45\sin\left[\frac{360(n+284)}{365}\right] \tag{3}$$

$$\omega_s = arcos\left[-\tan(\delta)\tan(\varphi)\right] \tag{4}$$

The average daily radiation on a tilted surface (H_T) can be expressed by

$$H_T = RH \tag{5}$$

where R is defined as the ratio of the daily average radiation on a tilted surface to that on a horizontal surface for each month. R can be estimated by considering individually the beam, diffuse and reflected components of the radiation incidence on the tilted surface. Assuming the diffuse and reflected radiation to be isotropic, Liu and Jordan have proposed that R can be expressed as [42]

$$R = \left(1-\frac{H_d}{H}\right)R_b + H_d\left(\frac{1+\cos(s)}{2H}\right) + \rho\left(\frac{1-\cos(s)}{2}\right) \tag{6}$$

where R_b is the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each month, s the tilt of the surface from horizontal, and ρ ($=0.2$) the ground reflectance. H_d is the monthly average daily diffuse radiation and is given by Page [11]

$$H_d = H(1.00-1.13K_T) \tag{7}$$

It is noted that R_b is a function of the transmittance of the atmosphere which depends upon the atmospheric cloudiness, water vapor and particulate concentration. However, Liu and Jordan suggest that R_b can be estimated to be the ratio of extraterrestrial radiation on the tilted surface to that on a horizontal surface for the month [45]. For surfaces facing directly towards the equator

$$R_b = \frac{\cos(\varphi-s)\cos(\delta)\sin(\omega_s') + \omega_s'(\pi/180)\sin(\varphi-s)\sin(\delta)}{\cos(\varphi)\cos(\delta)\sin(\omega_s) + \omega_s(\pi/180)\sin(\varphi)\sin(\delta)} \tag{8}$$

where ω_s' is the sunset hour angle for the tilted surface and is given by

$$\omega_s' = \min\left\{\omega_s, arcos\left[-\tan(\varphi-s)\tan(\delta)\right]\right\} \tag{9}$$

4. Results and discussion

The present study was carried out for the eight main provinces of Turkey. The above formulae were used to calculate the monthly average daily total radiation on a south facing surface as the tilt angle was changed from 0° to 90° in steps of 1°. The optimum angle for tilted surface was calculated by searching for the values of which the daily total solar radiation was at a maximum for a specific period. Therefore, the optimum tilt angle was found for certain provinces of Turkey.

Information for the provinces considered in this study is given in Table 1. The recommended average day and declination for each month are presented in Table 2 [46]. Table 3 shows the measured monthly average daily global solar radiation for these provinces of Turkey. It also shows the calculated diffuse and extraterrestrial solar radiation values, clearness index, optimum tilt-angle and monthly average daily global radiation on optimum tilted surface in eight provinces of Turkey. It was found that the optimum tilt angle changed between 0° (June and July) and 60° (December) for Adana, 0° (June) and 60° (December) for Ankara, 0° (June ad July) and 61° (December) for Diyarbakir, 0° (June) and 61° (December) for Izmir and Trabzon, 0° (June) and 65° (December) for Erzurum, 0° (June) and 59° (December) for Istanbul and 0° (June) and 62° (December) for Samsun throughout a year. The monthly average daily solar radiation availabilities of tilted surfaces for the provinces of Turkey are presented in Fig. 1. Fig. 2 shows monthly average daily solar radiations for the determined tilt angles when the solar collector is tilted at the optimum angle, the seasonal average, the yearly average, the latitude and the latitude ± 15. General correlations developed for the optimum tilt angles of the solar collectors used in Turkey are given in Fig. 3. Also, the results of this study are compared with the studies carried out by Ulgen [28] and Gunerhan and Hepbasli [29] in the city of Izmir in Fig. 4. As can be seen from Fig. 4, the results obtained from each study are in general agreement.

The values of the monthly average daily global solar radiation for the provinces of Turkey were taken from the Turkish State Meteorological Service for the period between 1991 and 2005. The optimum tilt angles were determined based on the experimental data of the monthly average global solar radiation.

Additionally, the correlation equations of the optimum tilt angle (S_{opt}) based on the declination factor are developed for Turkey in general. These correlation equations are given below

$$S_{opt} = 32.619-1.3629(\delta) \tag{10}$$

Table 1
Information for the provinces considered in the study.

Location	Longitude (*E*)	Latitude (*N*)	Elevation (*m*)
Adana	35.18	36.59	20
Ankara	32.53	39.57	894
Diyarbakır	40.12	37.55	660
Erzurum	41.16	39.55	1869
İstanbul	29.05	40.58	39
İzmir	27.10	38.24	25
Samsun	36.20	41.17	44
Trabzon	39.43	41.00	30

Table 2
Recommended average day and declination for each month [31].

Months	Day of the year	Date	Declination (δ)
January	17	17 Jan.	−20.92
February	47	16 Feb.	−13.29
March	75	16 Mar	−2.42
April	105	15 Apr.	9.41
May	135	15 May	18.79
June	162	11 June	23.09
July	198	17 July	21.18
August	228	16 Aug.	13.45
September	258	15 Sept.	2.22
October	288	15 Oct.	−9.60
November	318	14 Nov.	−18.91
December	344	10 Dec	−23.05

Table 3
Monthly average daily global (measured), diffuse (calculated) and extraterrestrial solar radiation on a horizontal surface, clearness index, optimum tilt-angle and monthly average daily global radiation on optimum tilted surface in eight provinces of Turkey (radiations are given as MJm^{-2}-day).

Location	Months	H	H_d	H_o	K_T	S_{opt} (°)	H_T
Adana	Jan.	7.78	3.79	17.12	0.45	58	12.43
	Feb.	10.82	4.87	22.23	0.49	50	14.80
	Mar.	14.58	6.20	28.65	0.51	36	16.93
	Apr.	17.74	7.62	35.12	0.51	19	18.38
	May	21.20	8.34	39.50	0.54	4	21.24
	Jun.	23.16	8.45	41.22	0.56	0	23.16
	Jul.	22.73	8.24	40.28	0.56	0	22.73
	Aug.	20.51	7.57	36.74	0.56	13	20.89
	Sep.	17.37	6.35	30.94	0.56	30	19.36
	Oct.	13.09	5.05	24.07	0.54	47	17.42
	Nov.	9.01	4.00	18.30	0.49	57	14.20
	Dec.	6.69	3.47	15.70	0.43	60	10.97
Ankara	Jan.	6.16	3.36	15.32	0.40	59	9.74
	Feb.	9.28	4.55	20.57	0.45	51	12.79
	Mar.	13.37	5.98	27.35	0.49	38	15.73
	Apr.	16.61	7.54	34.39	0.48	20	17.32
	May	20.87	8.35	39.32	0.53	6	20.95
	Jun.	23.35	8.44	41.32	0.57	0	23.35
	Jul.	23.88	7.87	40.26	0.59	2	23.88
	Aug.	21.25	7.17	36.24	0.59	16	21.83
	Sep.	17.45	5.92	29.85	0.58	33	19.96
	Oct.	11.86	4.80	22.53	0.53	49	16.16
	Nov.	7.36	3.66	16.53	0.45	58	11.60
	Dec.	5.11	2.98	13.88	0.37	60	8.20
Diyarbakir	Jan.	7.93	3.64	16.54	0.48	59	13.08
	Feb.	11.49	4.62	21.70	0.53	51	16.31
	Mar.	15.56	5.87	28.24	0.55	38	18.45
	Apr.	19.81	7.10	34.90	0.57	20	20.71
	May	24.14	7.45	39.45	0.61	5	24.20
	Jun.	27.85	6.61	41.26	0.67	0	27.85
	Jul.	26.94	6.58	40.29	0.67	0	26.94
	Aug.	24.39	6.02	36.59	0.67	15	25.03
	Sep.	20.71	4.87	30.60	0.68	33	23.92
	Oct.	14.75	4.33	23.58	0.63	50	20.77
	Nov.	9.78	3.68	17.73	0.55	59	16.40
	Dec.	6.83	3.34	15.12	0.45	61	11.60
Erzurum	Jan.	8.86	3.07	15.33	0.58	64	17.08
	Feb.	12.58	3.89	20.58	0.61	55	19.74
	Mar.	15.97	5.44	27.36	0.58	40	19.62
	Apr.	16.95	7.51	34.39	0.49	20	17.70
	May	19.89	8.52	39.32	0.51	6	19.96
	Jun.	23.17	8.49	41.32	0.56	0	23.17
	Jul.	23.19	8.10	40.26	0.58	2	23.20
	Aug.	21.21	7.18	36.24	0.59	16	21.79
	Sep.	17.25	5.99	29.86	0.58	33	19.69
	Oct.	12.57	4.65	22.54	0.56	50	17.49
	Nov.	8.79	3.51	16.54	0.53	61	15.16
	Dec.	7.03	3.01	13.89	0.51	65	13.44
Istanbul	Jan.	5.00	3.08	14.70	0.34	58	7.61
	Feb.	7.75	4.36	20.00	0.39	50	10.38
	Mar.	11.82	5.95	26.89	0.44	37	13.80
	Apr.	15.76	7.53	34.12	0.46	21	16.47
	May	20.44	8.41	39.24	0.52	8	20.54
	Jun.	22.92	8.56	41.34	0.55	0	22.92
	Jul.	22.69	8.23	40.23	0.56	3	22.71
	Aug.	19.15	7.65	36.05	0.53	16	19.68
	Sep.	15.79	6.23	29.46	0.54	34	18.01
	Oct.	10.17	4.86	21.99	0.46	48	13.57
	Nov.	6.16	3.47	15.93	0.39	57	9.45
	Dec.	4.20	2.70	13.26	0.32	59	6.55
Izmir	Jan.	7.58	3.55	16.13	0.47	60	12.58
	Feb.	10.42	4.66	21.32	0.49	51	14.50
	Mar.	14.78	5.94	27.94	0.53	38	17.48
	Apr.	18.27	7.41	34.73	0.53	20	19.06
	May	22.39	8.02	39.41	0.57	6	22.45
	Jun.	25.31	7.78	41.28	0.61	0	25.31
	Jul.	24.90	7.51	40.28	0.62	1	24.90
	Aug.	22.35	6.87	36.47	0.61	15	22.91
	Sep.	18.64	5.70	30.35	0.61	33	21.28
	Oct.	13.35	4.68	23.22	0.57	49	18.41
	Nov.	8.83	3.74	17.32	0.51	59	14.45

Table 3 (*continued*)

Location	Months	H	H_d	H_o	K_T	S_{opt} (°)	H_T
	Dec.	6.20	3.24	14.69	0.42	61	10.34
Samsun	Jan.	5.73	3.14	14.34	0.40	60	9.39
	Feb.	8.28	4.34	19.66	0.42	51	11.41
	Mar.	11.38	5.88	26.62	0.43	37	13.20
	Apr.	15.33	7.51	33.95	0.45	21	16.01
	May	19.44	8.54	39.19	0.50	7	19.53
	Jun.	22.59	8.64	41.35	0.55	0	22.59
	Jul.	23.00	8.14	40.21	0.57	3	23.02
	Aug.	19.96	7.43	35.93	0.56	17	20.56
	Sep.	15.14	6.28	29.23	0.52	33	17.17
	Oct.	10.00	4.79	21.68	0.46	48	13.31
	Nov.	6.87	3.45	15.57	0.44	59	11.19
	Dec.	4.90	2.80	12.90	0.38	62	8.34
Trabzon	Jan.	5.29	3.10	14.45	0.37	59	8.27
	Feb.	7.81	4.32	19.76	0.39	50	10.49
	Mar.	10.52	5.84	26.70	0.39	35	11.99
	Apr.	13.62	7.46	34.00	0.40	19	14.11
	May	16.57	8.66	39.21	0.42	7	16.63
	Jun.	18.40	9.15	41.34	0.45	0	18.40
	Jul.	16.71	8.87	40.22	0.42	3	16.72
	Aug.	14.44	7.89	35.97	0.40	14	14.69
	Sep.	12.32	6.47	29.30	0.42	30	13.51
	Oct.	8.84	4.78	21.77	0.41	46	11.26
	Nov.	6.12	3.42	15.67	0.39	57	9.35
	Dec.	4.56	2.75	13.01	0.35	61	7.41

$$S_{opt} = 34.784 - 1.3621(\delta) - 0.0081(\delta)^2 \tag{11}$$

$$S_{opt} = 34.783 - 1.4317(\delta) - 0.0081(\delta)^2 + 0.0002(\delta)^3 \tag{12}$$

The values of the optimum tilt angle calculated from Eqs. (10)–(12) are given in Table 4. In addition, it is noticeable that the optimum tilt angle for the month of June is negative; the negative sign determines the orientation of the solar collector, which means that the solar collector is faced towards the north. A positive sign indicates that the solar collector is directed towards the south [27].

Table 5 shows the statistical test results of the obtained correlation equations. It can be observed that the agreement is good among the results obtained by Eqs. (10)–(12). The optimum tilt angles to be used in winter (December, January and February), in spring (March, April, and May), in summer (June, July and August) and in autumn (September, October and November) for each province of Turkey are given in Table 6. Also, the values of latitude ± 15 and the yearly averages are presented in Table 6. These will be the optimum fixed tilt angles throughout the year. The values of the yearly total global radiation (in MJ/m^2-year) on a tilted surfaces in the eight provinces of Turkey are given in Table 7 when the tilt angles of the solar collector are adjusted by taking into account optimum, seasonal, yearly average, latitude, lat.+15 and lat.−15.

5. Statistical methods

In the literature, numerous statistical methods have been used to compare the models of solar radiation estimation. In the present study, the model equations are evaluated by using the most widely used statistical indicators which are Mean Bias Error (MBE), Root Mean Square Error (RMSE), t-statistic (t-stat) and correlation coefficient (r) [7–9,47–55].

5.1. Mean bias error (MBE)

The MBE can be calculated using the following equation [47]

$$\mathrm{MBE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - x_i) \tag{13}$$

where x_i is the i-th measured value, y_i the i-th calculated value, respectively and n is the total number of observations.

The MBE provides information on the long-term performance of an equation. A positive value gives the average amount of overestimation in the estimated values and vice versa. The smaller the value of MBE, the greater the performance of the equation.

5.2. Root mean square error (RMSE)

The RMSE can be computed using the following equation [47]

$$\mathrm{RMSE} = \left[\frac{1}{n}\sum_{i=1}^{n}(y_i - x_i)^2\right]^{1/2} \tag{14}$$

The RMSE provides information on the short-term performance of an equation. The value of the RMSE is always positive, representing zero in the ideal case.

5.3. t-statistic method (t-stat)

The t-statistic is defined as follows [48]

$$t\text{-}stat = \left(\frac{(n-1)\mathrm{MBE}^2}{\mathrm{RMSE}^2 - \mathrm{MBE}^2}\right)^{1/2} \tag{15}$$

The smaller the value of t-stat, the greater the performance of the model. In order to determine whether a model's estimates are statistically significant, one simply has to determine a t-critic value obtainable from standard statistical tables, i.e., $t_{\alpha/2}$ at the α level of significance and $(n-1)$ degrees-of-freedom. For the model's estimates to be judged statistically significant at the $1-\alpha$ confidence level, the calculated t-stat value must be less than the t-critic value [48].

5.4. Correlation coefficient (r)

The r value can be used to determine the linear relationship between the measured and estimated values, which can be calculated from the following equation [54]

$$r \equiv \left[(S_t - S_r)/S_t\right]^{1/2} \tag{16}$$

where S_t is the standard deviation and S_r is the deviation of the calculated value from the measured value. S_t and S_r are defined as follows:

$$S_t = \sum_{i=1}^{n} (x_a - x_i)^2 \tag{17}$$

$$S_r = \sum_{i=1}^{n} (x_i - y_i)^2 \tag{18}$$

where x_a is the average of the measured values and is given by

$$x_a = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{19}$$

In this study, the three models are compared on the basis of the above mentioned statistical error tests and, the accuracy of the estimated data for the three models is determined using these

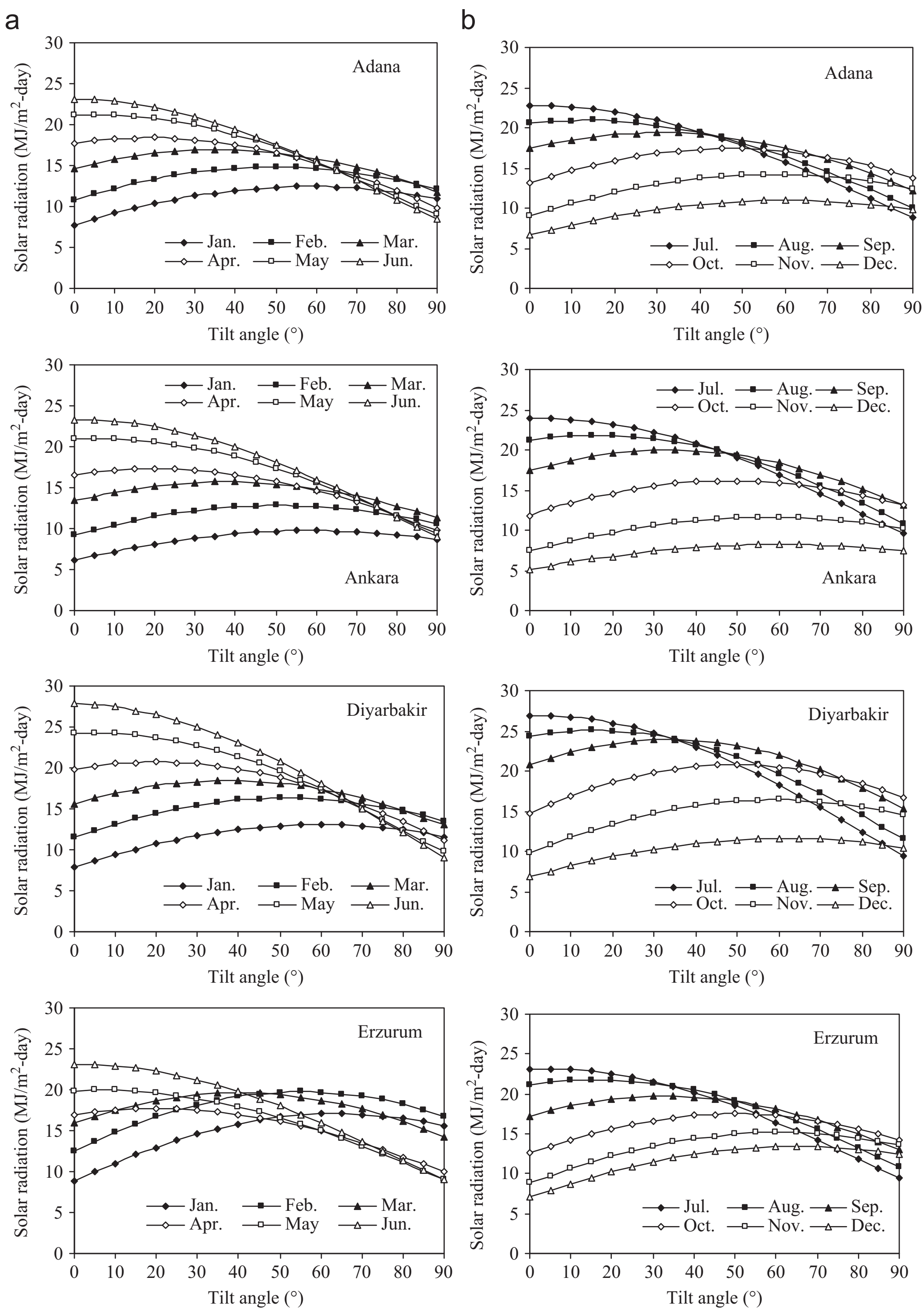

Fig. 1. Monthly average daily solar radiation availability of tilted surfaces, (a) January–June; (b) July–December.

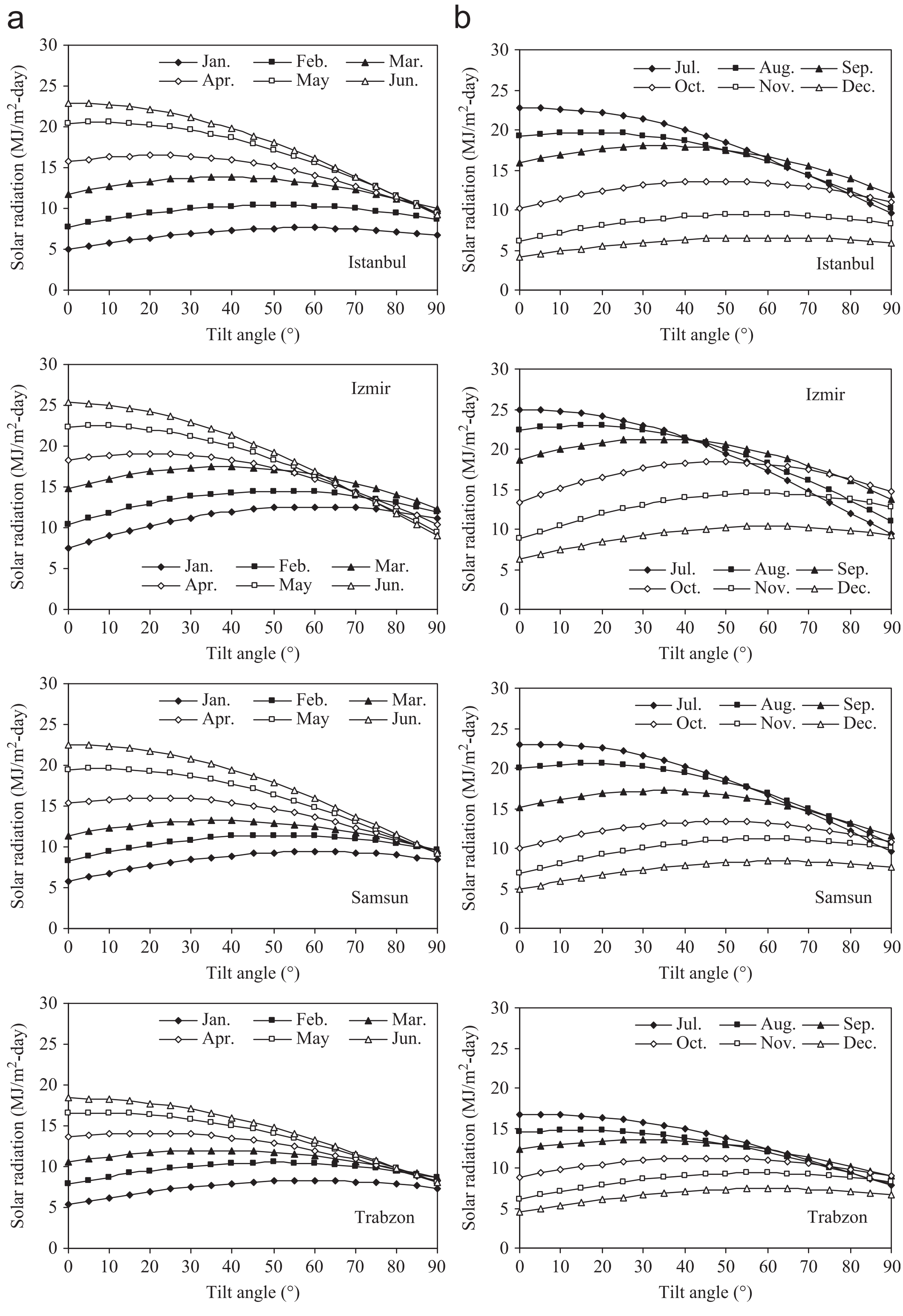

Fig. 1. (*continued*)

tests. For better data modeling, these statistics should be closer to zero, but r should approach to 1 as closely as possible.

6. Mathematical optimization formulation

The maximization problem of the average daily radiation on a tilted surface (H_T) can be expressed as an optimization problem as

$$\max H_T = RH$$

subject to

$$0 \leq s \leq 90 \tag{20}$$

where H_T is called the objective function to be maximized. R is the functions that depend on the design variable s, while H is a constant for a given month. Since the optimization problem given in Eq. (20) has only one design variable; i.e., s, the optimum results can be obtained by utilizing either the graphical

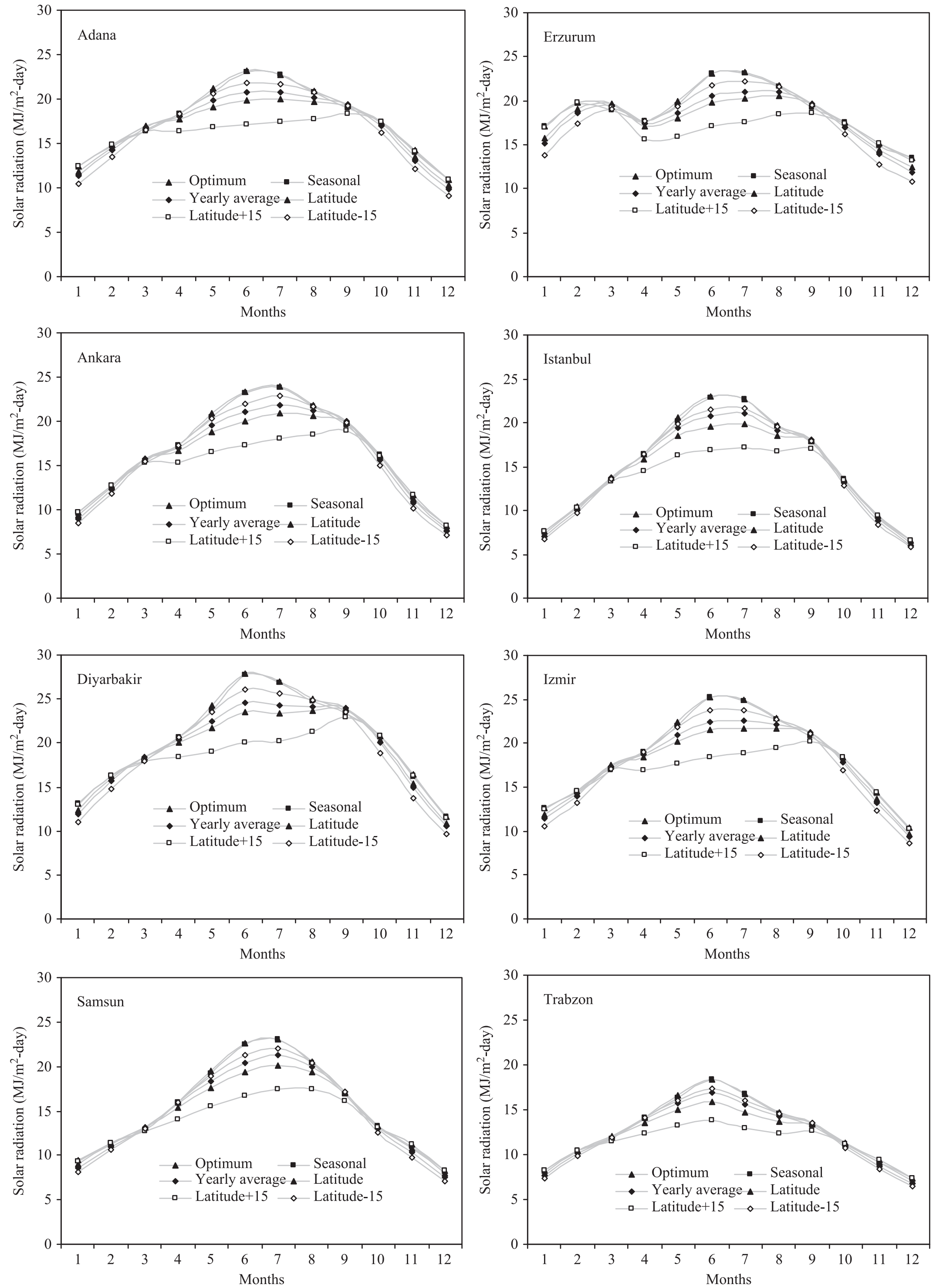

Fig. 2. Monthly average daily solar radiation for determined tilt angles.

optimization technique or one of the numerical optimization techniques for unconstrained optimization problems. The graphical representation of the objective function is given in Fig. 5.

The optimum value for s can be visually read from the figure. More precise results for s can be obtained using one of the numerical optimization methods. In this regard, a MATLAB [56] based program has been written, in which the MATLAB optimization toolbox was used to solve the optimization problem given in Eq. (20). An unconstrained optimization command in the MATLAB, *fminbnd*, was run, which is based on golden section search and parabolic interpolation [57]. The optimum value of s for January in the province of Adana, for example, was obtained as 58.32 and the corresponding value of H_T was 12.43. Consequently, it can be seen that the values obtained from the MATLAB optimization toolbox are identical to the values found by the methodology used in the study (Table 8).

7. Conclusions

The optimum tilt angles of the solar panels may be determined when the tilt angle is varied from 0° to 90° in steps of 1°. The monthly

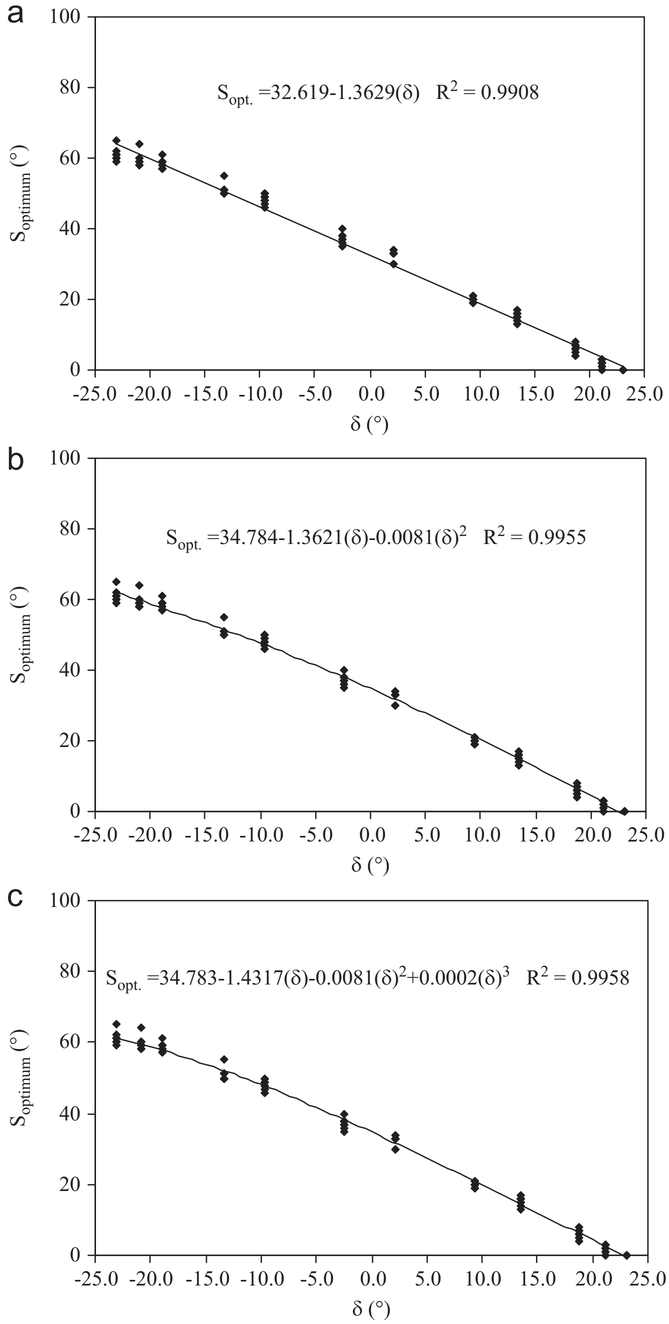

Fig. 3. General correlations developed for optimum tilt angles of solar collectors used in Turkey. (a) Linear, (b) Quadratic and (c) Third order polynomial.

and seasonal optimum tilt angles for the eight provinces having principal climatic conditions of Turkey were obtained and the general correlation equations giving the monthly optimum tilt angles for Turkey were developed.

According to the above results, the following conclusions can be drawn:

1. It was found that the optimum tilt angle changed between 0° (June) and 65° (December) throughout the year in Turkey. The optimum tilt angle decreased to a minimum of 0° in June and July for Adana and Diyarbakir, and only in June in other provinces. The monthly average daily total radiation at this angle was generally at a maximum.
2. The optimum tilt angles increased during the winter months and reached a maximum in December for all provinces. The

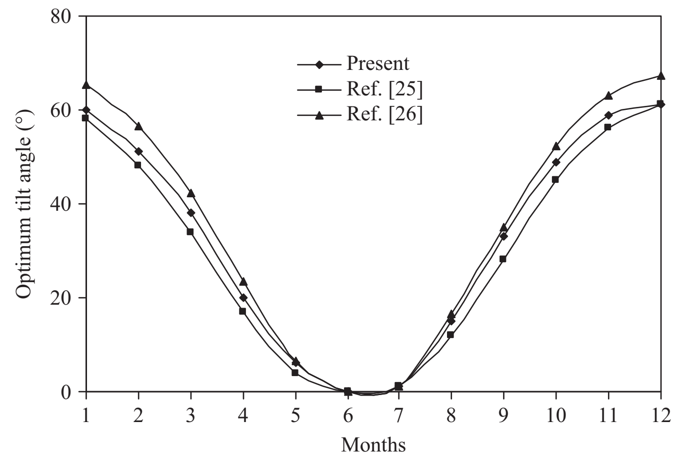

Fig. 4. Comparison of the results obtained in the literature for the province of Izmir.

Table 4
Values of the optimum tilt angle (in degrees) calculated from correlations obtained for Turkey.

Months	**Eq.** (10)	**Eq.** (11)	**Eq.** (12)
Jan.	61.1	59.7	59.4
Feb.	50.7	51.5	51.9
Mar.	35.9	38.0	38.2
Apr.	19.8	21.2	20.8
May	7.0	6.3	6.3
Jun.	1.1	−1.0	−0.1
Jul.	3.8	2.3	2.7
Aug.	14.3	15.0	14.5
Sep.	29.6	31.7	31.6
Oct.	45.7	47.1	47.6
Nov.	58.4	57.6	57.6
Dec.	64.0	61.9	61.0

Table 5
Statistical test results (t-critic=3.106 for α=0.01 and k=12) according to the correlation equations obtained in general of Turkey.

Location	Eq. no	MBE	RMSE	*t*-statistic	*r*
Adana	(10)	1.458	2.182	2.978	0.9952
	(11)	1.456	1.754	4.937	0.9969
	(12)	1.461	1.659	6.154	0.9972
Ankara	(10)	−0.042	2.165	0.064	0.9952
	(11)	−0.044	1.054	0.139	0.9989
	(12)	−0.039	0.893	0.147	0.9992
Diyarbakir	(10)	0.041	2.391	0.057	0.9945
	(11)	0.039	1.382	0.094	0.9982
	(12)	0.044	1.316	0.111	0.9983
Erzurum	(10)	−1.709	2.728	2.665	0.9934
	(11)	−1.711	2.404	3.359	0.9949
	(12)	−1.706	2.462	3.187	0.9946
Istanbul	(10)	0.041	2.248	0.208	0.9935
	(11)	0.039	1.477	0.088	0.9976
	(12)	0.044	1.369	0.106	0.9979
Izmir	(10)	−0.125	2.004	0.208	0.9961
	(11)	−0.127	1.017	0.418	0.9990
	(12)	−0.123	0.963	0.427	0.9991
Samsun	(10)	−0.542	1.698	1.117	0.9971
	(11)	−0.544	0.983	2.203	0.9990
	(12)	−0.539	1.094	1.881	0.9988
Trabzon	(10)	0.875	1.288	3.067	0.9983
	(11)	0.873	1.449	2.501	0.9978
	(12)	0.877	1.394	2.686	0.9980

Table 6
Latitude ±15, seasonal and yearly average tilt angles for provinces of Turkey.

Location	Lat. − 15°	Lat. + 15°	Spring	Summer	Autumn	Winter	Yearly average
Adana	22.01	52.01	19.67	4.33	44.67	56.00	31.2
Ankara	24.56	54.56	21.33	6.00	46.67	56.67	32.7
Diyarbakır	22.55	52.55	21.00	5.00	47.33	57.00	32.6
Erzurum	24.55	54.55	22.00	6.00	48.00	61.33	34.3
İstanbul	26.01	56.01	22.00	6.33	46.33	55.67	32.6
İzmir	23.25	53.25	21.33	5.33	47.00	57.33	32.8
Samsun	26.17	56.17	21.67	6.67	46.67	57.67	33.2
Trabzon	26.00	56.00	20.33	5.67	44.33	56.67	31.8
Average	**24.39**	**54.39**	**21.17**	**5.67**	**46.38**	**57.29**	**32.56**

Table 7
Yearly total global radiation (as MJm^{-2}-year) on a tilted surface in eight provinces of Turkey according to the adjusted as optimum, seasonal, yearly, latitude, lat. + 15 and lat. − 15 tilt angles of the solar collector.

Location	Optimum	Seasonal	Yearly	Latitude	Lat. + 15	Lat. − 15
Adana	6483.11	6418.55	6140.32	6097.49	5788.87	6118.71
Ankara	6149.00	6091.89	5847.49	5780.00	5437.35	5852.79
Diyarbakır	7481.56	7401.19	7052.27	7006.02	6635.73	7027.77
Erzurum	6954.04	6876.39	6535.64	6516.38	6246.40	6466.71
İstanbul	5544.78	5496.67	5293.50	5201.99	4857.26	5312.17
İzmir	6824.67	6753.94	6453.27	6404.08	6059.82	6439.60
Samsun	5667.54	5616.77	5386.48	5310.44	4983.71	5395.53
Trabzon	4661.38	4623.18	4450.55	4386.25	4131.21	4453.94

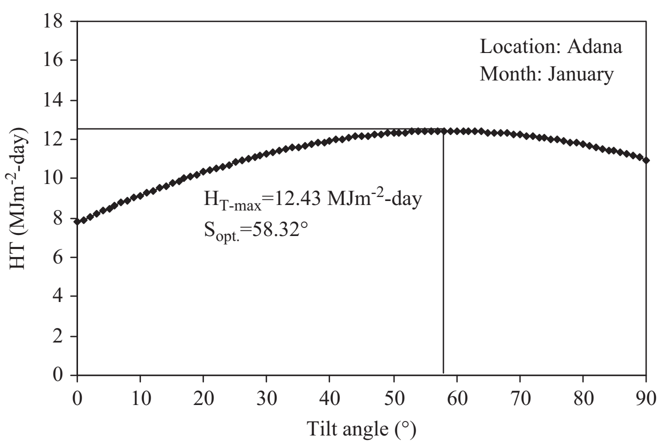

Fig. 5. Graphical representation of the objective function (H_T versus s).

Table 8
Values of S_{opt} and H_T of each province for January.

Location	Mathematical optimization		Method used in the study	
	S_{opt} (°)	H_T	S_{opt} (°)	H_T
Adana	58.32	12.43	58	12.43
Ankara	58.76	9.74	59	9.74
Diyarbakır	59.45	13.08	59	13.08
Erzurum	63.94	17.08	64	17.08
İstanbul	57.82	7.61	58	7.61
İzmir	59.86	12.58	60	12.58
Samsun	60.36	9.39	60	9.39
Trabzon	58.76	8.27	59	8.27

monthly solar energy collected in December was 10.97, 8.20, 11.60, 13.44, 6.55, 10.34, 8.34 and 7.41 MJm^{-2}-day for Adana, Ankara, Diyarbakir, Erzurum, Istanbul, Izmir, Samsun and Trabzon, respectively. However, these values of the monthly solar energy were the smallest of those obtained during the year.

3. When the monthly optimum tilt angle was used, (i) the largest amount of monthly solar energy collected during the year were 23.16 (June), 23.88 (July), 27.85 (June), 23.20 (July), 22.92 (June), 25.31 (June), 23.02 (July) and 18.40 (June) in MJm^{-2}-day, (ii) the yearly collected solar energy were 6483.11, 6149.00, 7481.56, 6954.04, 5554.78, 6824.67, 5667.54 and 4661.38 in MJ/m^2-year for Adana, Ankara, Diyarbakir, Erzurum, Istanbul, Izmir, Samsun and Trabzon, respectively. This value (6824.67 MJm^{-2}-year) was calculated for Izmir above and found to be 6820.36 MJm^{-2}-year by Ref. [18].
4. When the monthly optimum tilt angles were used, the yearly amount of collected solar energy ranged from 5635.86 to 6483.11 in MJ/m^2-year for Adana. Therefore, the enhancement of the yearly collected solar energy was 15.03%. These percentage rates were 14.07, 16.63, 21.58, 12.22, 15.83, 14.16 and 13.00 for Ankara, Diyarbakir, Erzurum, Istanbul, Izmir, Samsun and Trabzon, respectively.
5. The yearly average optimum tilt angles for a south-facing solar collector were 31.2°, 32.7°, 32.6°, 34.3°, 32.6°, 32.8° and 33.2° in Adana, Ankara, Diyarbakir, Erzurum, Istanbul, Izmir, Samsun and Trabzon, respectively. When the yearly average tilt angles were used, the yearly collected solar energy were 6140.32, 5847.49, 7052.27, 6535.64, 5293.50, 6453.27, 5386.48 and 4450.55 in MJm^{-2}-year for Adana, Ankara, Diyarbakir, Erzurum, Istanbul, Izmir, Samsun and Trabzon, respectively.
6. When the seasonal tilt angles were used, the yearly collected solar energy was 6418.55, 6091.89, 7401.19, 6876.39, 5496.67, 6753.94, 5616.77 and 4623.18 in MJm^{-2}-year for Adana, Ankara, Diyarbakir, Erzurum, Istanbul, Izmir, Samsun and Trabzon, respectively. The optimum tilt angles for spring, summer, autumn and winter on average for Turkey were 21.17°, 5.67°, 46.48° and 57.29°, respectively.
7. The seasonal average tilt was calculated by finding the average value of the tilt angle for each season. The implementation of this requires the collector tilt to be changed four times a year. To increase the efficiency of solar collectors, the solar collector should be mounted at the monthly average tilt angle.
8. It is noted that the amount of collected solar energy for all provinces was generally in order of optimum, seasonally and yearly average (in Table 7).
9. The model Eqs. (10)–(12) obtained in the study can be used to estimate the optimum tilt angle of a solar collector in Turkey. According to the results of the statistical tests of the correlation coefficient (r), Eq. (12) generally gave the minimum errors when compared with Eqs. (10) and (11). Also, the equation having the best determination coefficients (R^2 = 0.9958) among the three models is Eq. (12).

10. The model equations giving the greatest results for each province differ according to the statistical methods (Table 5). Considering the MBE; the model equations giving the best results are Eqs. (11) and (12) for the provinces of Adana, Ankara, Diyarbakir, Erzurum, Istanbul, Izmir, Samsun and Trabzon, respectively. Considering the RMSE; the model equations giving the best results are the Eqs. (10)–(12) for the provinces of Adana, Ankara, Diyarbakir, Erzurum, Istanbul, Izmir, Samsun and Trabzon, respectively. Considering the t-statistic; all of the model equations are statistically significant excluding Eqs. (11) and (12) for both Adana and Erzurum.

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